Sequential Formation of Alliances in Survival Contests

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Abstract

We consider a sequential formation of alliances à la Bloch (1996) and Okada (1996) followed by a two-stage contest in which alliances …rst compete with each other, and then the members in the winning alliance compete again for an indivisible prize. In contrast to Konishi and Pan (2019) which adopted an open-membership game as the alliance formation process, alliances are allowed to limit their memberships (excludable alliances). We show that if members' e¤orts are strongly complementary to each other, there will be exactly two asymmetric alliances— the larger alliance is formed …rst and then the rest of the players form the smaller one. This result contrasts with the one under open membership, where moderate complementarity is necessary to support a two-alliance structure. It is also in stark contrast with Bloch et al. (2006), where they show that a grand coalition is formed in the same game if the prize is divisible and a binding contract is possible to avoid further con‡icts after an alliance wins the prize.

1 Introduction

In their in‡uential paper, Esteban and Sákovics (2003) consider a three-person strategic alliance formation in a Tullock contest model in which players compete for an indivisible prize, and demonstrate that an alliance involves strategic disadvantages (see also Konrad 2009). There are two main disadvantageous forces against forming an alliance: First, if an alliance is formed, there will be

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an additional contest that dissipates the members' rents even if the alliance wins the first race. Because of this rent-dissipation effect, the members of the alliance have lower valuations for winning in the first race, reducing their efforts and the winning probability. Second, even without the rent-dissipation problem, if the winning prize is shared equally, there are still free-riding incentives for the alliance members to reduce efforts, and consequently, the winning probability. As a result, they conclude that it is hard to materialize strategic alliances in a Tullock contest model.[1,2] Konrad (2009) points out that these disincentive effects are not specific to Tullock contest models— they also appear in first price all-pay auctions.

In a companion paper, Konishi and Pan (2019), we provide a simple solution for this alliance paradox by using a CES effort aggregator function to introduce complementarity in efforts (see Kolmer and Rommeswinkel 2013).[3] We assume that each individual member's marginal effort cost is constant in order to limit the benefits of forming an alliance to effort complementarity only. In that paper, we model an alliance formation process as an open-membership coalition formation game. In stage 1, players form alliances by an open-membership game (see Yi, 1997, and Bogomolnaia and Jackson, 2002). In stage 2, alliances compete in a contest with each other, and in stage 3, the winning alliance members compete in the standard Tullock contest for the indivisible prize. We show that when the complementarity parameter in CES function is small, there are spin-off incentives for alliance members, while when the complementarity parameter is large, players want to join a bigger alliance,

[1] Konrad (2004) considers an asymmetric all-pay auction game with exogenously determined hierarchical tournament structure, and shows that the highest valuation player might not have a chance to become the final winner depending on the hierarchical structure. In contrast, Konrad (2012) consider an alliance formation problem in the case where players with homogeneus valuations play an all-pay auction game while their budgets for bidding are private information. He shows that alliances always have merging incentives, and the grand alliance emerges.

[2] Wärneryd (1998) shows that forming alliances and competing in a multi-stage competition reduce wasteful competition and increase total welfare. This resource saving effect is difficult to realize due to the disadvantageous effect on alliances when members' individual efforts are perfectly substitutable.

[3] Complementarity in efforts within a group in Esteban and Ray (2011) is more subtle. They analyze the conflict between two ethnic groups by assuming that players have heterogeneous opportunity costs of financial and human opportunity costs, and they can contribute financially to a conflict or they can directly participate as activists. They show that opportunity cost heterogeneity in a group increases the level of conflicts. Their result can be interpreted that an increase in complementarity within groups intensifies group competition.

and end up with a trivial grand alliance.[4] They show that for intermediate values of the CES complementarity parameter, there exists a unique nontrivial two-alliance equilibrium.

In contrast, in this paper, we use Bloch's (1996) and Okada's (1996) sequential coalition (alliance) formation game (along the line of a noncooperative coalition bargaining game in Chatterjee, et al. 1993). Although the open-membership game in Konishi and Pan (2019) is widely used in coalition formation games, the non-excludability – that is, players are allowed to freely choose their alliance without being excluded – may not reflect the nature of alliance formation in situations such as a local public good economy.

The results in open-membership and sequential coalition formation games are quite different. In an open-membership game, if effort complementarity is higher than a critical value, belonging to a larger alliance becomes strongly preferable, despite the fact that there will be negative congestion effects, which encourages all players to form a grand coalition. This is a prisoners' dilemma phenomenon. In contrast, with sequential coalition formation, a coalition is able to avoid becoming too large, although it also needs to think about the response from the rest of the players in their strategic interactions. Somewhat interestingly, there will again be two alliances in equilibrium, but for this we need strong effort complementarity. Note that due to excludability, even if complementarity is very strong, the grand alliance will not emerge in equilibrium. Thus, although the two-alliance result seems similar to the one in the open-membership case, they have no obvious relationship. Indeed, the parameter ranges to have two-equilibrium results in these two games have no intersection with each other, and two coalitions are similar in their sizes in the open-membership game, but are quite asymmetric in the sequential coalition formation game. We further show that the first alliance is larger than the second, and the members of the former receive higher payoffs than the latter. This property assures that the alliance structure is robust in the protocol: that is, we obtain the same alliance structure in Bloch's deterministic protocol and in Okada's random protocol. Note that Bloch (1997) and Yi (1997) provide a set of sufficient conditions under which two coalitions are formed in a sequential alliance formation game, but these conditions and our conditions are independent of each other. Moreover, we get the two-alliance result only when effort complementarity is large enough.

We also provide numerical examples for different values of the CES effort complementarity parameter under a small number of players (ten players). We show that there will be no alliance if is small, but as goes up the sizes of alliances increase. Once passes a certain threshold value, there will be only two (asymmetric) alliances in equilibrium, and every player participates in alliances as we have shown in our main theorem.

The rest of the paper is organized as follows. In the next subsection, we review the relevant literature. Section 2 introduces the model, and Sections 3 and 4 investigate subgames in stages 3 and 2, respectively. Section 5 presents results on equilibrium alliance structures, and Section 6 provides numerical examples. Section 7 concludes.

1.1 Literature Review

Since we provide a general literature review in our companion paper (Konishi and Pan 2019), we will concentrate on the games that determine an alliance structure. In the companion paper, we used so-called open-membership game where all players can move freely without being excluded from alliances.[5] However, depending on the nature of alliances we consider, we may want to see how equilibrium alliance structure is a¤ected by allowing exclusive memberships of alliances.

Although we can think of di¤erent ways to introduce "excludability" of alliance memberships in an alliance formation game (see Hart and Kurz 1983, and Bloch 1997), the most popular way in the literature is to extend Rubinstein's two-person noncooperative bargaining game to a sequential coalition formation game: Chatterjee et al. (1993), Bloch (1996), Okada (1996), and Ray and Vohra (1999), among others. Although their games di¤er in the methods of choosing the proposers (following di¤erent protocols), the procedures for forming coalitions are the same. At each stage, a proposer proposes a coalition she belongs to, and ask the members of the coalition whether or not they accept the o¤er. If every member accepts the o¤er, then the coalition is formed, and the leftover players continue to form coalitions by the same procedure. If any of the members of a proposed coalition rejects the o¤er, the coalition is not formed, and a new proposer is speci…ed by the protocol.

In the context of contests, Bloch et al. (2006) generalize the model substantially to analyze the stability of the grand alliance in di¤erent alliance formation games, including a sequential coalition formation game in Bloch (1996). Sánchez-Páges (2007a) explores di¤erent types of stability concepts

[5] Baik and Lee (1997, 2001) use open-membership games to describe alliance formation in endogenizing the alliance structure in Nitzan's (1991) game with endogenous group sharing rules.

including sequential coalition formation games in alliance formation in con-and a time discount factor $\in$; applies to the …nal payo¤. The process continues until there is no player left and $\{S_1; S_2; ::::; S\}$ is formed.

We introduce potential bene…ts for players who belong to an alliance—complementarity in aggregating e¤orts by all alliance members. That is, if player i belongs to alliance j ; ::::; J with $S \subset N$ as the set of members, and these members make e¤orts e $_2$, then the aggregated e¤ort of alliance j, E, is described by a CES aggregator function

$$E \quad @ \overset{0}{} \sum_{2} e^{1} \overset{1}{A}^{\frac{1}{1-}} ; \tag{1}$$

where $\in$; is a parameter that describes the degree of complementarity: if it is a linear function, and if it is a Cobb-Douglas function. Thus, as goes up, the complementarity of members' e¤orts increases.

Candidate i in alliance j decides how much e¤ort e to contribute to her alliance j. The winning probabilities of an alliance is a Tullock-style contest. That is, an alliance j's "winning probability" given its members' e¤orts is

$$p \quad \frac{E}{\sum_{2} E} : \tag{2}$$

An indivisible prize is valued as $V >$, which is common to all players. Since the prize is indivisible, one player in the winning alliance in the second stage must be selected as the …nal winner in the third-stage contest.

In the third-stage competition, we assume that a Tullock contest takes place within the winning alliance S. Denoting the second-stage e¤ort as e, the winning probability of player $i \in S$ is

$$p \quad \frac{e}{\sum_{2} e} \tag{3}$$

Formally, an alliance structure is a partition of the set of players N, $\{S_1; ::::; S\}$; where each alliance j consists of a set of players S and $\cup_2 S$ N, and $S \cap S$ $\emptyset$ for any $j; j^0 \in \{ ; ::::; J\}$ with $j \; / \; j^0$. Since we assume that players are ex-ante homogenous, we also call $\{n_1; :::; n\}$ an alliance structure with n $|S|$ for all j ; ::::; J. Our three-stage dynamic contest game with sequential alliance formation is summarized as:

Stage 1. In round j ; ; ::::; one player is selected as a proposer with equal probability among all active players in the round j, N, where N_1 N.[7]

[7] This is the random proposer protocol put forth by Okada (1996). Bloch (1996) uses a deterministic protocol, but the results we obtain in these two setups are the same if e¤ort complementarity is high enough.

The selected player proposes an alliance $S \subseteq N$. All other players in S either accept or reject the proposal sequentially. If all other players in the alliance S accept the proposal, S is formed and removed from the process, and j round starts with the remaining players $N_{+1} \quad N \setminus S$. Otherwise, payo¤ discounts by $\in \; ;$ apply to all players, the round r starts with $N_{+1} \quad N$ by the same rule. The process continues until there is no player left and $\{S_1; S_2; ::::; S\}$ is formed.[8]

Stage 2. All players $i \in N$ choose e¤ort $e \in {}_+$ simultaneously, knowing the aggregated e¤ort of her alliance is (1). The inter-alliance contest is a Tullock contest with winning probabilities equal to (2).

Stage 3. All members of the winning alliance S choose e¤ort $e \in {}_+$ simultaneously. The ultimate winner is selected by a simple Tullock contest with winning probabilities equal to (3).

We use standard subgame perfect Nash equilibrium as the solution of this dynamic game. We consider equilibria in pure strategies only. We will analyze this game by backward induction.

3 Equilibrium

3.1 Stage 3: Final Contest within the Winning Alliance

In the third stage, all members in the winning alliance S in the …rst stage engage in a Tullock contest by exerting e¤ort $e \geq$. Thus, player i's winning probability is

$$p \quad \frac{e}{\sum_{2} e}:$$

For any player i in the winning group j, the expect payo¤ in stage 3 is

$$V$$

Since players are homogeneous, $p \quad - p \quad \frac{1}{2}$ is the same for all i in the winning group j. Then, we have the following proposition.

Proposition 1. Suppose that the winning alliance of the first stage has size n . Then, the second-stage equilibrium strategy and payoff are

$$e \quad \frac{n \; -}{n^2} V \text{ and } V \quad \frac{V}{n} \quad - \frac{n \; -}{n} \quad \frac{V}{n^2}$$

3.2 Stage 2: Contest between Alliances

Consider an inter-alliance contest problem. Without loss of generality, we reorder any alliance structure from the first stage so that $n_1 \geq n_2 \geq \cdots \geq n_{*}$. From Proposition 1, we know that for a given size of alliance n the payoff of intra-alliance contest is determined by $V \quad \frac{}{2}$. In the companion paper, Konishi and Pan (2019) have the following result.

Theorem 1. (Konishi and Pan, 2019) There exists a unique equilibrium in the second stage for any partition of players $\{n_1; \ldots; n_{*}\}$ characterized by $j \in \{ ; \ldots; J \}$ such that $p >$ (active alliance) for all $j \leq j$, while p (inactive alliance) for all $j > j$. Moreover, the members of alliance $j \quad ; \ldots; J$ obtain payoff

$$u \quad \begin{cases} \left[\frac{1}{2} \quad - j - \frac{\frac{2-3}{1-}}{\sum_{\ell=1}^{*} \frac{2-3}{1-}} \right. & \#5(0)a \\ & \end{cases}$$

3.3 Stage 1: Alliance Structures under Sequential Coalition Formation

Here, we consider a sequential coalition formation game with exclusive alliances a la Bloch (1996) and Okada (1996). The main results are as follows.

Theorem 2. For any N, there is a N such that, for all $\geq$ a N , there are only two alliances in equilibrium. All players belong to one of the two and

$$g\ x; \bar{x}\ J \qquad \frac{J\ \frac{1}{\ } -\ J -\ \ \frac{1}{\ }}{x^2\ J\ \frac{1}{\ }\ \ \frac{1}{\ }}$$

We have the following result.

Lemma 1. Suppose that $J \geq$ alliances with their average size $\bar{x}$ have been formed and remain active even with the entry of the J alliance. Then, (i) $\frac{(\ \ ;\ +1)}{\ } <$ for all x and $\bar{x}$, and (ii) $\frac{(\ \ ;\ +1)}{\ } >$ holds for all $\frac{1}{\ }\bar{x} \leq$ $- \leq \frac{(2+\ )\ \ 4}{2}$ when $\geq$. Moreover, if $- < \frac{1}{\ }$, then even if the J th alliance with size x enters, it cannot be active.

The implications of this lemma are listed in the following corollaries.

Corollary 1. When $> \frac{4}{\ }$, then the best response of the J th alliance satis…es $x > \bar{x}$ knowing that the alliance with a higher winning probability dominates the loss from sharing with a larger group.

Lemma 6. Suppose that among J formed alliances, J $\geq$ of them have the largest size x , and x < $-$ $\sum_{=1}$ x < x . For $\geq$ N for some N , we have u x $\frac{1}{}$; x J > u x; x J for all x $\leq$ x , and u x ; x $\frac{1}{}$ J < u $\frac{1}{2}$ $-$ $\sum_{=1}$ x $\frac{1}{}$; $\frac{1}{2}$ $-$ $\sum_{=1}$ x $-$ $\frac{1}{}$. That is, the bene…ts of belonging to a larger alliance with a higher winning probability dominates the losses of sharing with a larger group.

Proof of Theorem 2. We can rename a N by the maximum of the original N , N ,] N , and N . Let a N be that corresponds to a N : By the sequence of the lemmas above, we consider the second mover's best or better responses.

1. Suppose that $x_1 \geq \frac{1}{2}$. By Lemma 1, x_2 $- x_1$ is the best response.
2. Suppose that $\frac{1}{3} \leq x_1 < \frac{1}{2}$. Suppose that $x_2 \leq \frac{1 \quad 1}{2}$. We will show that forming multiple same-size alliances is dominated by forming an alliance of size x_1 $\frac{1}{}$. Suppose that two or more size-x_2 alliances are formed after a size-x_1 alliance. In this case, $x_2 \leq x_1$ holds. By Lemma 3, having only one size-x_2 alliance is generally better than forming multiple of them. Since $x_2 \leq x_1$, calling x_2 is dominated by calling x_1 by Lemma 1. But Lemma 4 suggests that for the second mover calling x_1 $\frac{1}{}$ dominates calling x_1, since Lemma 2 implies that there will be only two active alliances if x_1 is called. NNNNI[]0d0J0.478+

this behavior by the $J-$ th alliance, the $J-$ th alliance can call a little more than one half of the set of players who do not belong to alliances 1 to $J-$ (Lemma 6). Then, only the $J-$ th and the $J-$ th alliances will remain active, and alliance 1 gets zero payo¤ (the $J-$ th alliance is formed by all of the rest of the players by Lemma 1). Thus, this case cannot be an equilibrium as well.

Hence, only case 1 can happen in equilibrium, and there are only two alliances in equilibrium, all players belong to one of the alliances, and the …rst alliance is larger than the second.

Remark. Since $x_1 > x_2$ holds with $u\ x_1; x_2 > u\ x_2; x_1$ in equilibrium, there will not be any delay in forming coalitions. That is, the same outcome would realize independent of the protocol.

4 Examples with Small Population

For our analytical result, we will consider the cases of relatively low complementarity parameter with a small number of players N . The complementarity parameter value $\geq \frac{6}{}$

: ; ; . If the second alliance calls a size 3 alliance, then the third alliance will be size 3, and their payo¤s for ; ; are : ; : ; : . Thus, the second mover will call a size 5, and the payo¤s for ; are ; : .

5. The …rst mover calls a size 3 alliance. If the second mover calls a size 3, then the rest form a size 4, and this is not bene…cial for the second mover (see above). If she calls a size 4, then ; ; realizes with : ; : ; : . If she calls a size 5, then ; realizes, leaving an inactive size 2 alliance with payo¤s : ; : . So, her best response is to call a size 4 alliance.

6. The …rst mover calls a size 2 alliance. Then, the second mover calls a size 5 alliance, making the …rst mover's alliance inactive. The payo¤s for ; are (: ; : .

In summary, the …rst mover calls size 6 alliance. The …rst two alliances' payo¤s from ; are : ; : .

4.2 Case 2: $\frac{5}{6}$ or

When $\frac{5}{6}$, the general pattern is similar to the case of $\frac{6}{7}$, except for

4.3 Case 3: Smaller s

When $\frac{4}{5}$ ( ), the situation is the same as in the $\frac{5}{6}$ case. The equilibrium (active) alliance structure for this case is ; . How about for an even smaller ? When $\frac{3}{4}$ ( ), we have an (active) equilibrium alliance structure ; ; , achieving payoffs : . Note that this number is higher than the payoff from ; , : . With this low complementarity, even if the first mover calls a size 3 alliance, the second mover does not benefit by calling a size 4 or 5 alliance. Having a large alliance just intensifies the subsequent fight, and ; ; realizes.

When $\frac{2}{3}$ ( ), the equilibrium alliance structure is ; ; ; ; with payoffs : . There will be no further spinoff for this N , since calling a one person alliance increases the number of alliances, which is harmful to the player (an independent player gets $\frac{1}{36}$ < : from ; ; ; ; ;). However, if N goes up, all alliances are resolved, going back to the standard Tullock competition.

5 Concluding Remarks

In this paper, we consider an alliance formation game in Tullock contests when efforts by the members of an alliance are complementary to each other. In order to illustrate excludability of alliance memberships, we use Bloch's noncooperative game of sequential coalition formation (1996). Unlike in an open-membership game analyzed in the companion paper (Konishi and Pan 2019), strong complementarity does not mean a grand alliance, since alliances can exclude outsiders by limiting membership. We show that there will be only two asymmetric alliances in which (i) all players belong to one of them, and (ii) the first alliance is larger than the second alliance, when effort complementarity is large enough. With a small population example, we show that (i) there can be more than two alliances in equilibrium, and (ii) there can be fringe inactive players in equilibrium when effort complementarity is not too strong. These results sheds light on the role of exclusivity in forming alliances in the context of contest games.

Appendix

We collect all the proofs of lemmas in the text.

Proof of Lemma 1. We start by di¤erentiating f and g with respect to x:

$$\frac{@f \; x; x \; J}{@x} \quad \frac{- \; J \frac{1}{+1} J \frac{1}{+1}}{J \frac{1}{} \quad \frac{1}{}^{2}} <$$

and

$$\frac{@g \; x; x \; J}{@x} \quad \frac{- \; J \frac{1}{+1} \; J \frac{1}{} \quad \frac{1}{} \quad J \frac{1}{} - \; J - \quad \frac{1}{} \quad J \frac{1}{+1}}{x^{2} \; J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{- \; J \frac{1}{+1} \quad J \frac{1}{} \quad \frac{1}{} \; - \; J \frac{1}{} - \; J - \quad \frac{1}{}}{x^{2} \; J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{- \; J \frac{1}{+1} \times J \frac{1}{}}{x^{2} \; J \frac{1}{} \quad \frac{1}{}^{2}} <$$

These imply that $\frac{(\; ; \; +1)}{} <$: i.e., a coalition's payo¤ declines if other active coalitions' sizes increase.

Di¤erentiating f and g with respect to x, we have

$$\frac{@f \; x; x \; J}{@x} \quad \frac{J}{N} \frac{\frac{1}{+2} \; J \frac{1}{} \quad \frac{1}{} \quad \frac{1}{+1} - \frac{1}{+1}}{J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{J}{N} \frac{J \frac{1}{+2} \frac{1}{} \quad \frac{1}{2 \; +2}}{J \frac{1}{} \quad \frac{1}{}^{2}} >$$

$$\frac{@g \; x; x \; J}{@x} \quad \frac{J - \quad \frac{}{+1} x^{2} \; J \frac{1}{} \quad \frac{1}{} - J \frac{1}{} - \; J - \quad \frac{1}{} \quad x \; J \frac{1}{} \quad \frac{1}{} - x^{2} \frac{}{+1}}{x^{4} \; J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{J - \quad \frac{}{+1} x \; J \frac{1}{} \quad \frac{1}{} - J \frac{1}{} - \; J - \quad \frac{1}{} \quad J \frac{1}{} \quad \frac{1}{} - x \frac{}{+1}}{x^{3} \; J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{J - \quad - - \quad J \frac{1}{} - \; J - \quad \frac{1}{} \quad J \frac{1}{} \quad \frac{1}{} \quad J \frac{1}{} - \; J - \quad \frac{1}{} \; -}{x^{3} \; J \frac{1}{} \quad \frac{1}{}^{2}}$$

$$\frac{J - \quad \frac{1}{} - \; J \frac{1}{} \quad J \frac{1}{} \quad \frac{1}{} \quad J \frac{1}{} - \; J - \quad \frac{1}{} \; -}{x^{3} \; J \frac{1}{} \quad \frac{1}{}^{2}} \tag{4}$$

Thus, $\frac{(\; ; \; +1)}{} >$ (thus $\frac{(\; ; \; +1)}{} >$) holds if we have

$$\frac{J -}{J} \leq \frac{x}{x} \leq \frac{J -}{J}$$

We can relax the su¢ cient upperbound slightly:

respectively. We have

$$J x_2$$

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