

## ALGEBRA QUALIFYING EXAM { SPRING 2017

**Problem 1.** Prove that an Artinian ring has finitely many maximal ideals.

**Problem 2.** Let  $\mathbb{F}$  be a finite field with  $|\mathbb{F}| = q$ . Consider the subgroup

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \mid a \in \mathbb{F}^* ; b \in \mathbb{F} \right\} < \text{GL}_2(\mathbb{F}) :$$

Show that for any prime  $p$  dividing  $q - 1$ , the number of Sylow  $p$ -subgroups of  $G$  is  $q$ .

**Problem 3.** Let  $R$  be a UFD and  $a, b$  be coprime elements in  $R$ . For all  $i \geq 0$ , compute

$$\text{Tor}_i^{R/(ab)}(R/(a); R/(b)) :$$

**Problem 4.** Let  $F$  be a field, and  $D$  be an integral domain containing  $F$ . Suppose  $D$  is finite dimensional as a vector space over  $F$ . For each  $x \in D$ , define the  $F$ -linear transformation  $T_x: D \rightarrow D$  by  $T_x(y) = xy$ .

(a) Prove that  $F$